OC Problem Set 2

Friday, May 8, 2015

Problem 1: Maxwell's Equations and Wave Equation

In a medium which is non-conducting and free of electric space charges ($\sigma = 0$, $\vec{J} = 0$, $\vec{\rho} = 0$) Maxwell's equations are:

$$\mathrm{curl}\vec{H} = \frac{\mathrm{d}\vec{D}}{\mathrm{d}t} \qquad \mathrm{curl}\vec{E} = -\frac{\mathrm{d}\vec{B}}{\mathrm{d}t}$$

$$\mathrm{div}\,\vec{B} = 0 \qquad \mathrm{div}\,\vec{D} = 0$$

with $\vec{D} = \varepsilon\vec{E}$ and $\vec{B} = \mu\vec{H}$.

a) Part a) is optional:
Assume a medium with $\varepsilon(\vec{r}) = \varepsilon_0\varepsilon_r(\vec{r})$ and $\mu(\vec{r}) = \mu_0\mu_r(\vec{r})$. Derive the wave equations for the electric and magnetic field starting from Maxwell's equations. Use Cartesian coordinates.
Hints:
$$\mathrm{curl}\,\mathrm{curl}\vec{A} = \mathrm{grad}\,\mathrm{div}\,\vec{A} - \nabla^2\vec{A}$$
$$\mathrm{curl}(\alpha\vec{A}) = \alpha\,\mathrm{curl}\vec{A} + \mathrm{grad}(\alpha)\times\vec{A}$$
$$\mathrm{div}(\alpha\vec{A}) = \alpha\,\mathrm{div}\,\vec{A} + \vec{A}\cdot\mathrm{grad}(\alpha)$$

Now assume a lossless, homogeneous, isotropic and unmagnetic medium ($\mu = \mu_0$ and $\varepsilon = \mathrm{const}$ be valid in a limited frequency range). Simplify the equations derived above, and introduce the refractive index n and the vacuum speed of light c. The result corresponds to equation 2.12 from the lecture notes. (Compare lecture slides 57/58 from April, 14)

b) Consider a monochromatic electromagnetic wave of the form
$$\Psi(t,x,y,z) = \tilde{\Psi}(k_x,k_y)\mathrm{e}^{-\mathrm{j}(k_x x + k_y y + k_z z)}\mathrm{e}^{\mathrm{j}\omega t}.$$
Substitute this ansatz into the differential equation derived in a), or to equation 2.12 from the lecture notes, respectively. Derive a relationship between the components of the propagation vector components k_x, k_y, k_z and the angular frequency ω. Show, that the phase fronts $\vec{k}\cdot\vec{r} = \mathrm{const}$ are planes.

Problem 2: Phase velocity and group velocity

A light pulse at wavelength $\lambda = 1500\,\mathrm{nm}$ propagates over a length of 6 km in a medium with refractive index $n = 1.5$. At the same time, a second light pulse propagates in parallel in free space. The dispersion of the medium at $\lambda = 1500\,\mathrm{nm}$ is given by

$$\frac{\mathrm{d}n}{\mathrm{d}\lambda} = -1\times10^{-5}\,\mathrm{nm}^{-1}.$$

a) Which time delay would result after 6 km due to the different *phase* velocities?

b) Which time delay would result after 6 km due to the different *group* velocities?

c) Which time delay (according to a) or b)) is actually observed?

For questions and suggestions on the OC tutorial please contact:

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